TYJ – 01 MATHEMATICS SOLUTION
TEST – 19, DATE 01 AUGUST 2019

31. (c) Given expression

$= 2[1 + {}^9C_2(3\sqrt{2}x)^2 + {}^9C_4(3\sqrt{2}x)^4$

$+ {}^9C_6(3\sqrt{2}x)^6 + {}^9C_8(3\sqrt{2}x)^8]$

$\therefore$ The number of non-zero terms is 5.

32. (c) $(1+100)^{100} = 1 + 100.100 + \frac{100.99}{1.2}.(100)^2$

$+\frac{100.99.98}{1.2.3}(100)^3 + ...$

$(101)^{100} - 1 = 100.100\left[1 + \frac{100.99}{1.2} + \frac{100.99.98}{1.2.3}.100 + ...\right]$

From above it is clear that,

$(101)^{100} - 1$ is divisible by $(100)^2 = 10000$

33. (d) $T_3 = {}^nC_2(x)^{n-2}\left(-\frac{1}{2x}\right)^2$ and $T_4 = {}^nC_3(x)^{n-3}\left(-\frac{1}{2x}\right)^3$

But according to the condition,

$\frac{-n(n-1)\times 3\times 2\times 1\times 8}{n(n-1)(n-2)\times 2\times 1\times 4} = \frac{1}{2} \Rightarrow n = -10$

34. (c) ${}^{20}C_{r-1} = {}^{20}C_{r+3} \Rightarrow 20 - r + 1 = r + 3 \Rightarrow r = 9$.

35. (c) $\frac{1}{6} = \frac{{}^nC_6(2^{1/3})^{n-6}(3^{-1/3})^6}{{}^nC_{n-6}(2^{1/3})^6(3^{-1/3})^{n-6}}$ or $6^{-1} = 6^{-4}.6^{n/3} = 6^{n/3-4}$

$\therefore \frac{n}{3} - 4 = -1 \Rightarrow n = 9.$

36. (a) We have $T_{r+1} = {}^{21}C_r\left(\sqrt[3]{\frac{a}{\sqrt{b}}}\right)^{21-r}\left(\sqrt{\frac{b}{\sqrt[3]{a}}}\right)^r$

$= {}^{21}C_r a^{7-(r/2)} b^{(2/3)r-(7/2)}$

Since the powers of *a* and *b* are the same, therefore

$7 - \frac{r}{2} = \frac{2}{3}r - \frac{7}{2} \Rightarrow r = 9$

37. (b) We have $(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + ...$

By hypothesis, $\frac{m(m-1)}{2}x^2 = -\frac{1}{8}x^2$

$\Rightarrow 4m^2 - 4m = -1 \Rightarrow (2m-1)^2 = 0 \Rightarrow m = \frac{1}{2}$.

38. (c) $T_1 = {}^nC_0 = 1$ .....(i)

$T_2 = {}^nC_1 ax = 6x$ .....(ii)

$T_3 = {}^nC_2(ax)^2 = 16x^2$ .....(iii)

From (ii), $\frac{n!}{(n-1)!}a = 6 \Rightarrow na = 6$ .....(iv)

From (iii), $\frac{n(n-1)}{2}a^2 = 16$ .....(v)

Only (c) is satisfying equation (iv) and (v).

39. (a) In the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$, the general term is $T_{r+1} = {}^{10}C_r\left(\frac{x}{2}\right)^{10-r}.\left(-\frac{3}{x^2}\right)^r$

$= {}^{10}C_r(-1)^r.\frac{3^r}{2^{10-r}}x^{10-r-2r}$

Here, the exponent of x is $10 - 3r = 4 \Rightarrow r = 2$

$\therefore\ T_{2+1} = {}^{10}C_2\left(\frac{x}{2}\right)^8\left(-\frac{3}{x^2}\right)^2 = \frac{10.9}{1.2}.\frac{1}{2^8}.3^2.x^4$

$= \frac{405}{256}x^4$

$\therefore$ The required coefficient $= \frac{405}{256}$.

40. (d) $(1+x+x^3+x^4)^{10} = (1+x)^{10}(1+x^3)^{10}$

$= (1+{}^{10}C_1.x+{}^{10}C_2.x^2+.....)(1+{}^{10}C_1.x^3+{}^{10}C_2.x^6+.....)$

$\therefore$ Coefficient of $x^4 = {}^{10}C_1.{}^{10}C_1+{}^{10}C_4 = 310$.

41. (b) Here $T_{r+1} = {}^9C_r\left(\frac{x^2}{2}\right)^{9-r}\left(\frac{-2}{x}\right)^r$

$= {}^9C_r\frac{x^{18-3r}(-2)^r}{2^{9-r}}$, this contains x^{-9} if $18-3r=-9$ *i.e.* if $r=9$. Coefficient of x^{-9}

$= {}^9C_9\frac{(-2)^9}{2^0} = -2^9 = -512$.

42. (d) $T_{r+1} = {}^9C_r(3)^{9-r}(ax)^r = {}^9C_r(3)^{9-r}a^rx^r$

$\therefore$ Coefficient of $x^r = {}^9C_r3^{9-r}a^r$

Hence, coefficient of $x^2 = {}^9C_23^{9-2}a^2$ and coefficient of $x^3 = {}^9C_33^{9-3}a^3$

So, we must have ${}^9C_23^7a^2 = {}^9C_33^6a^3$

$\Rightarrow \frac{9.8}{1.2}.3 = \frac{9.8.7}{1.2.3}.a \Rightarrow a = \frac{9}{7}$.

43. (b) $(10-r)\left(\frac{1}{2}\right)+r(-2)=0 \Rightarrow 5-\frac{r}{2}-2r=0 \Rightarrow r=2$

So the term independent of x

$= {}^{10}C_2\times\left(\frac{1}{3}\right)^4\left(\frac{3}{2}\right)^2 = \frac{10\times9}{2\times1}\times\frac{1}{3\times3\times2\times2} = \frac{5}{4}$

44. (a) In the expansion of $\left(\frac{3x^2}{2}+\frac{1}{3x}\right)^9$, the general term is $T_{r+1} = {}^9C_r.\left(\frac{3x^2}{2}\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$= {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^rx^{18-3r}$

For the term independent of x, $18-3r=0 \Rightarrow r=6$

This gives the independent term

$T_{6+1} = {}^9C_6\left(\frac{3}{2}\right)^{9-6}\left(-\frac{1}{3}\right)^6 = {}^9C_3.\frac{1}{6^3}$

45. (c) $1(6-r)+(-1)r=0 \Rightarrow r=3$, therefore fourth term will be independent of x *i.e.*

${}^6C_3(2x)^3\left(\frac{1}{3x}\right)^3 = 20\times8\times\frac{1}{27} = \frac{160}{27}$